

PZMC Math Challenge Questions - Summer 2021

Instructions. Try as many of the following questions as you can. There is no time limit, but you should do these questions **on your own without any help from people, books, internet, or any other sources**. We are more interested in how you approach the questions and how you communicate your reasoning than in how many correct answers you obtain. For each solution you submit, please include a clear and complete explanation of your answer. Please typeset your answers. Send any questions you have to apacelli@williams.edu.

1. If 29 bakery staffers can decorate 29 cupcakes in 29 minutes, then how many bakery staffers are required to decorate 87 cupcakes in 87 minutes?

2. What is the negation of the following sentence?

In every beautiful beach house in the northeast, the master bedroom has a view of waves crashing against the rocks or the kitchen has navy blue cabinets.

3. Mr. Z, construction boss, has 3 boxes of building supplies, containing nails and screws. One is labeled ‘nails,’ another is labeled ‘screws,’ and the last is labeled ‘nails and screws.’ We know that a tired employee labeled each box incorrectly. Taking only one item out of one box, how can we label the boxes correctly for Mr. Z?

4. Reuben wrote a large number on the board and asked the students to tell about the divisors of the number one by one.

The 1st student said, “The number is divisible by 2.” The 2nd student said, “The number is divisible by 3.” The 3rd student said, “The number is divisible by 4.” . . . (and so on) The 30th student said, “The number is divisible by 31.

Reuben then commented that exactly two students, who spoke consecutively, spoke incorrectly.

Which two students spoke incorrectly, and how do you know?

5. There’s a box of five hats: two blue and three white. Andy, Kate, and James each place a hat on his or her head, while blindfolded. One by one, each child removes his blindfold and (without using a mirror) gets one opportunity to guess the color of the hat on his own head. If any of the three guesses correctly, everyone gets to go to the park! Andy, Kate, and James are each very logical, and know that the others are as well.

First, James removes his blindfold. He sees the hats that the others are wearing, but admits that he is unable to discern his own hat color.

Next, Kate removes her blindfold, and sadly reveals that she too is not able to determine the color of her own hat.

Finally, Andy pipes up and says “I can answer with my blindfold on! I know what color hat I am wearing.”

What color is Andy’s hat, and how does he know? Explain fully.

6. There are 100 doors, and 100 people lined up in a row. All the doors are closed. The first person goes through and opens all the doors. The second person goes through and changes the position (open to closed or closed to open) of every second door. The third person goes through and changes the position of every third door. And so on. After all 100 people have gone through the doors, which doors are open and which are closed? Explain fully.

7. You are blindfolded, and on the table in front of you are a number of shiny smooth discs. They are each about the size of a quarter; one side is purple and the other side green. You can certainly determine how many discs there are altogether, but you can’t tell whether a given disc is purple side up or green side up. Your friend Seth tells you (once) how many are purple side up. Your challenge is to separate the discs into two collections, flipping over whichever discs you like, so that each collection has the same number of purple side up discs. Explain how you complete the challenge, and how you know you’re successful.

8) Write down a mathematically interesting extension to one of the problems above that one could investigate. Do not do the problem, just pose the question.