

PZMC Math Challenge Questions - Summer 2022

Instructions. Try as many of the following questions as you can. There is no time limit, but you should do these questions **on your own without any help from people, books, internet, or any other sources**. We are more interested in how you approach the questions and how you communicate your reasoning than in how many correct answers you obtain. For each solution you submit, please include a clear and complete explanation of your answer. Please typeset your answers. Send any questions you have to pzmath@pzmath.com.

1. There's a small village at the base of a mountain with a population of 7777. There are at least 2 people in the village that do not have dogs. Also, given any three people in the village, at least one of the three has a dog. Do we have enough information to determine exactly how many people in the village have dogs? Explain why not, or find the exact number. Explain your reasoning.

2. What is the negation of the following sentence? In other words, what is the related statement that is true if and only if the following statement is false, and vice versa?

Every day in 2021, I did the New York Times Spelling Bee or I went for a long run with my dog.

3. Mr. Z, construction boss, has 3 boxes of building supplies, containing nails and screws. One is labeled 'nails,' another is labeled 'screws,' and the last is labeled 'nails and screws.' We know that a tired employee labeled each box incorrectly. Taking only one item out of one box, how can we label the boxes correctly for Mr. Z?

4. Reuben wrote a large number on the board and asked the students to tell about the divisors of the number one by one.

The 1st student said, "The number is divisible by 2." The 2nd student said, "The number is divisible by 3." The 3rd student said, "The number is divisible by 4." . . . (and so on) The 30th student said, "The number is divisible by 31."

Reuben then commented that exactly two students, who spoke consecutively, spoke incorrectly.

Which two students spoke incorrectly, and how do you know?

5. There's a box of five hats: two blue and three white. Andy, Kate, and James each place a hat on his or her head, while blindfolded. One by one, each child removes his blindfold and (without using a mirror) gets one opportunity to guess the color of the hat on his own head. If any of the three guesses correctly, everyone gets to go to the park! Andy, Kate, and James are each very logical, and know that the others are as well.

First, James removes his blindfold. He sees the hats that the others are wearing, but admits that he is unable to discern his own hat color.

Next, Kate removes her blindfold, and sadly reveals that she too is not able to determine the color of her own hat.

Finally, Andy pipes up and says “I can answer with my blindfold on! I know what color hat I am wearing.”

What color is Andy’s hat, and how does he know? Explain fully.

6. You are blindfolded, and on the table in front of you are a number of shiny smooth discs. They are each about the size of a quarter; one side is purple and the other side green. You can certainly determine how many discs there are altogether, but you can’t tell whether a given disc is purple side up or green side up. Your friend Seth tells you (once) how many are purple side up. Your challenge is to separate the discs into two collections, flipping over whichever discs you like, so that each collection has the same number of purple side up discs. Explain how you complete the challenge, and how you know you’re successful.

7. Five pirates of different ages (A , B , C , D , and E oldest to youngest) have a treasure of 100 gold coins. On their ship, they decide to split the coins using this scheme:

The oldest pirate proposes how to share the coins, and ALL pirates (including the oldest) vote for or against it.

If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain. As pirates tend to be a bloodthirsty bunch, if a pirate would get the same number of coins if he voted for or against a proposal, he will vote against so that the pirate who proposed the plan will be thrown overboard.

A) Assuming that all 5 pirates are intelligent, rational, greedy, and do not wish to die, (and are rather good at math for pirates) what will happen?

B) Come up with another mathematically interesting extension to this problem that you could investigate. You don’t need to do the problem, only pose the question.